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Poke the Problem of Connecting Dots

Poke the Problem of Adding Numbers

Poke the Problem of Dividing Squares

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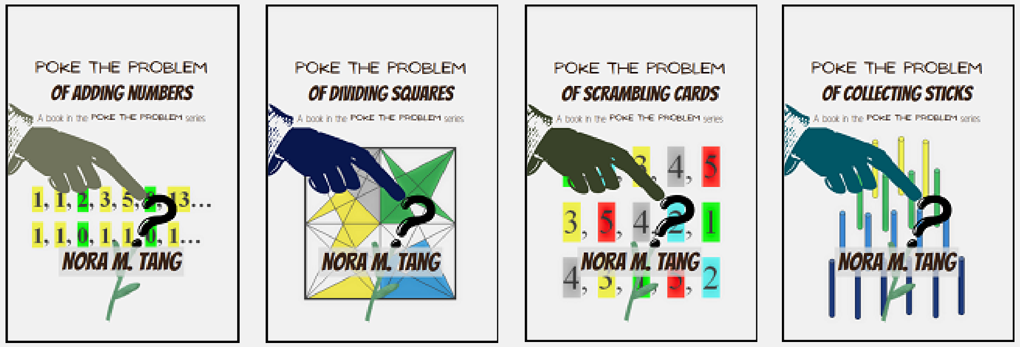

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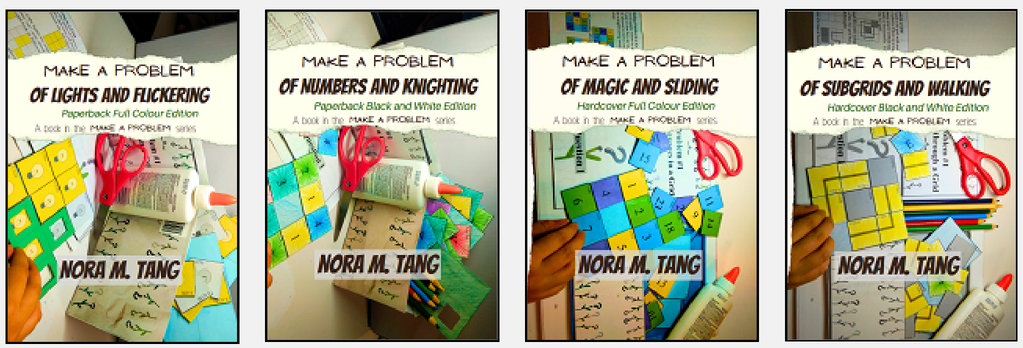

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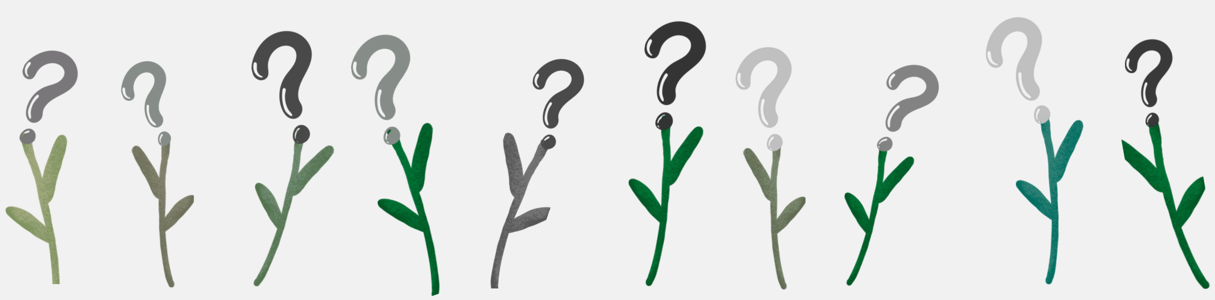
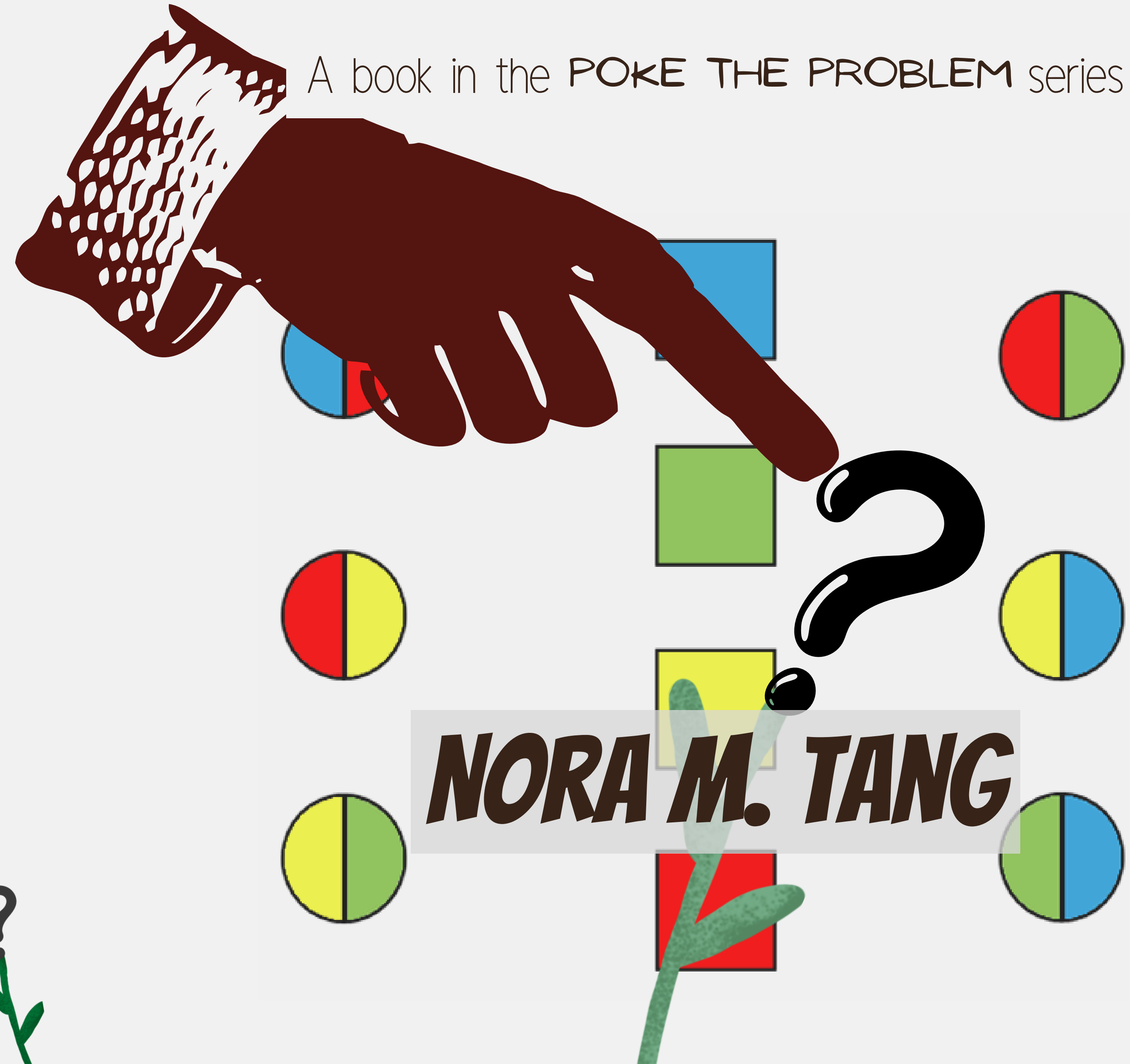


POKE THE PROBLEM

OF CONNECTING DOTS

A book in the POKE THE PROBLEM series



NORA M. TANG

Poke the Problem of Connecting Dots

A Book in the ***Poke the Problem*** Series

Nora M. Tang

ISBN-13: 978-1-989802-63-2

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Dedications:

For my daughters,

May you one day find yourselves in an elephant-less room surrounded by a boundless field of dandelions.

Table of Contents:

Preface

About a decade ago I had the chance to step back from my career in math education to start a family. From the time my young children were old enough to sit, I have been envious of language educators as children were surrounded by all the picture story books they may ever wish to read. The choices they have are endless, as are the lists of topics, styles, and authors to explore.

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Children are naturally curious, just as they are imaginative. Children can be excited by an interesting math question just as much as they can be by a story. Nor is there a lack of interesting problems math has to offer. Why, then, is there an absolute lack of interesting math problem books for children? More importantly, is this lack of exposure to interesting mathematics contributing to a child's relationship with the subject down the road?

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This book would not exist without the love and support of my husband, Victor, who has always done everything he can to hold space for my dreams and ideas, even when I couldn't see them clearly. Thank you for always being there for me.

Nora M. Tang

Revised April, 2024

0.

Before We Start

~*~

What you need to start enjoying problems in this book:

- a pen or pencil
- scrap paper
- a few minutes of free time
- a willingness to explore

There is *no time limit* for these problems. You are welcome to *use any calculator*. *The internet* and *other people's mathematical knowledge* are also within your allowed tools. You can even *change the question* if a different one seems more interesting to you, but be sure to record your new questions on the pages provided at the back of the book. In short, anything that helps you explore and understand is encouraged.

Mathematical knowledge needed to start exploring:

- none

Related mathematical topics to look up:

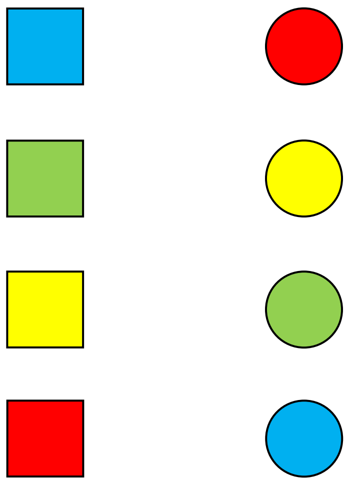
- combinations and permutations of distinct objects
- Euler's Polyhedron Formula
- cycle lengths of planar graphs

1.

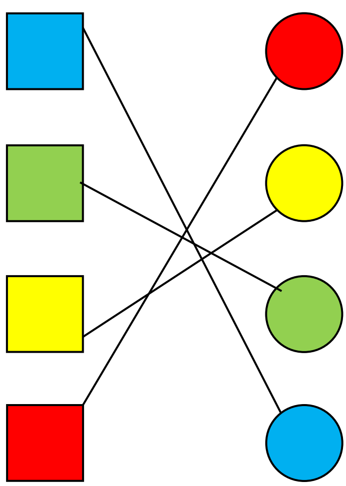
Connect the Dots

~*~

Let's play a game of Connecting Dots. The goal is to draw lines connecting every square to the circle that has the same colour without any of the lines crossing. For example, let's start here:



Note that connecting them with straight lines would result in the lines crossing. In other words, this is *not* a valid solution:

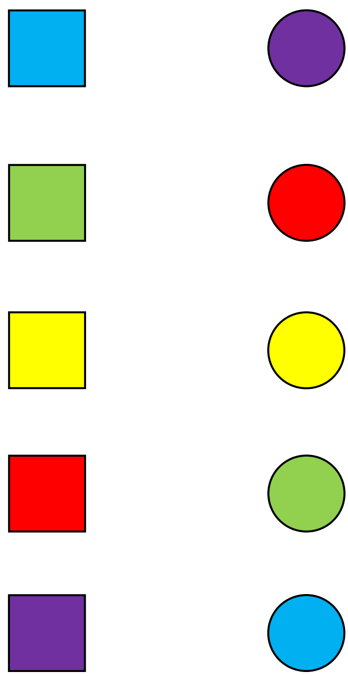


2.

Connect More Dots

~*~

Again, connect each square to the circle of the same colour without crossing lines:

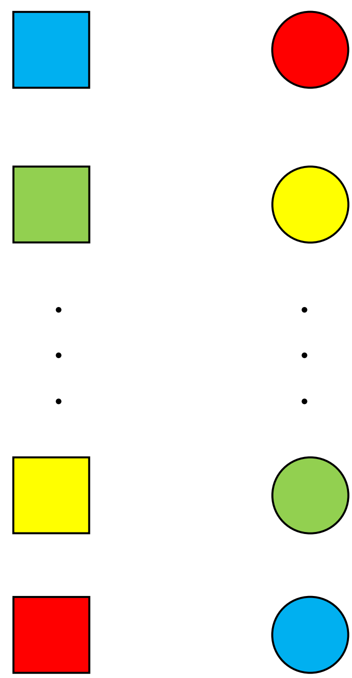


3.

Connect Many Dots

~*~

Is it always possible to connect the squares to their matching circles without crossing lines if the circles are in reverse order as the squares, regardless of how many colours there are?

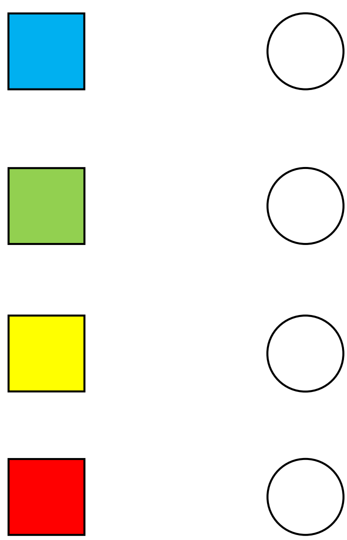


4.

Connect Rearranged Dots

~*~

Consider the same game of Connecting Dots with four colours. Is it always possible to connect the squares to their matching circles without crossing lines regardless of the order the four circles are arranged?

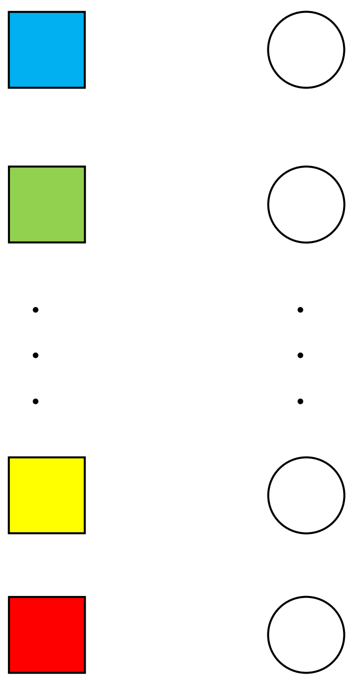


5.

Connect Many Rearranged Dots

~*~

Is it always possible to connect the squares to their matching circles without crossing lines, regardless of the number of colours and the order the circles are arranged?

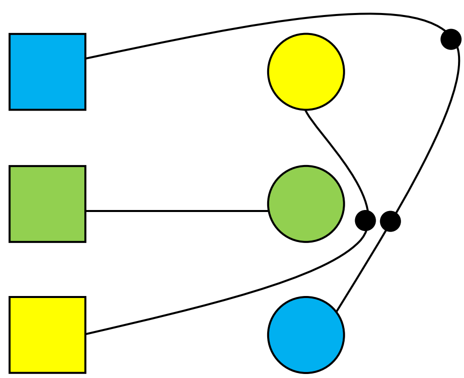


6.

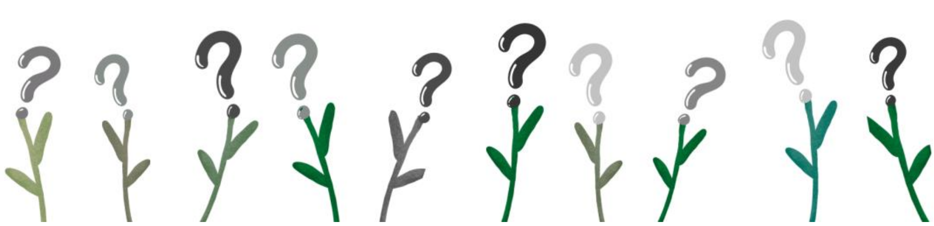
Loop the Dots

~*~

Here is a solution to the game of Connecting Dots with three colours. Notice there are two lines looping around the back of the green circle and one line looping around the back of the yellow circle.



How many solutions to this arrangement of shapes are there that have a maximum of one line looping around the back of each shape?

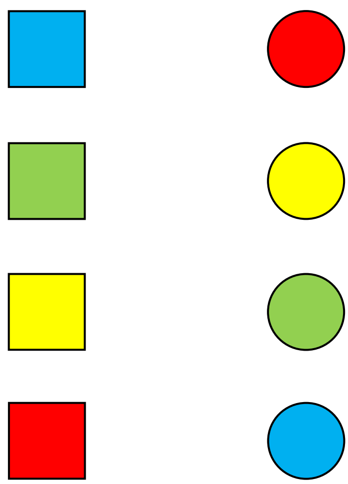


7.

Loop More Dots

~*~

How many solutions to this four colour game of Connecting Dots are there that have a maximum of one line looping around the back of each shape?

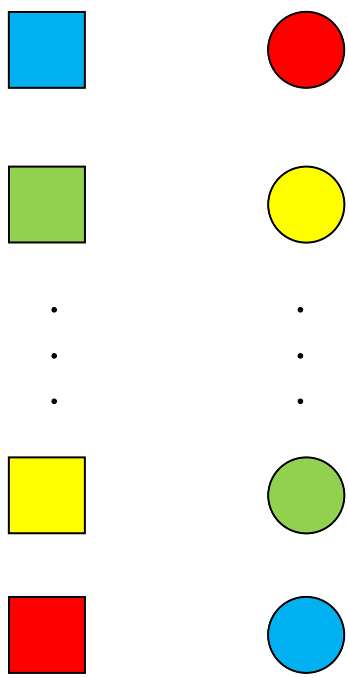


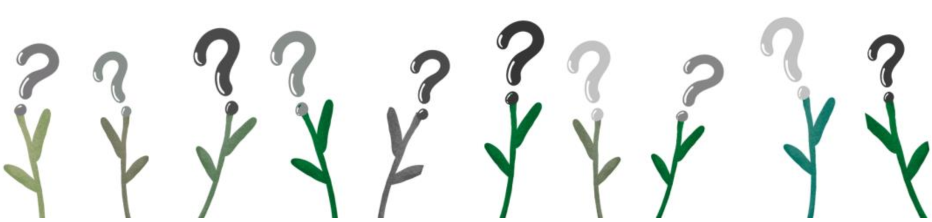
8.

Loop Many Dots

~*~

If a game of Connecting Dots has *n* colours, where *n* is a natural number, and the colours of the circles are arranged in reverse order as the squares, then how many lines looping around the back of each shape should we allow for this game to have a solution?



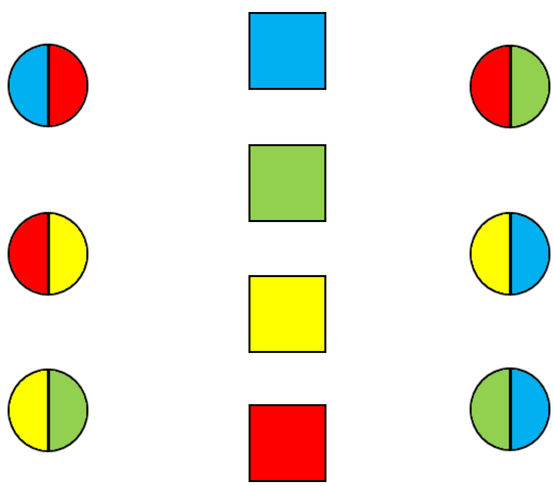


9.

Connect Double Dots

~*~

In the following game of Connecting Dots, each circle has two colours. Connect each circle to both squares that match its colours without crossing lines.

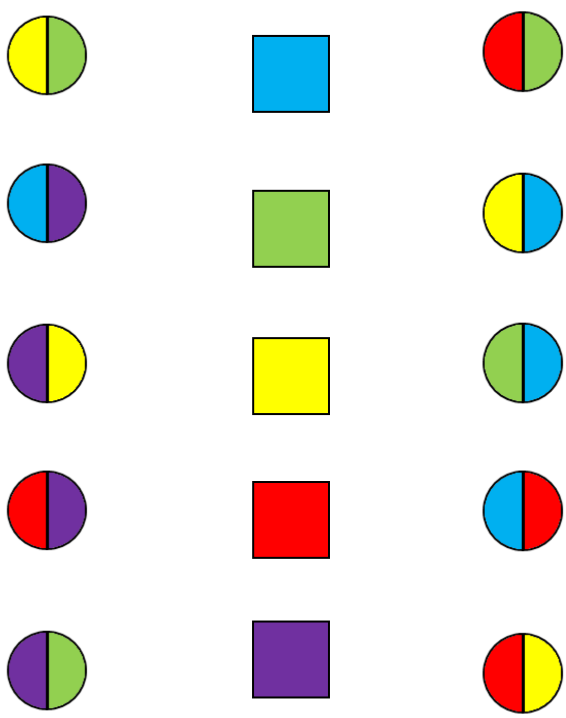


10.

Connect More Double Dots

~*~

Again, connect each circle to both squares that match its colours without crossing lines.

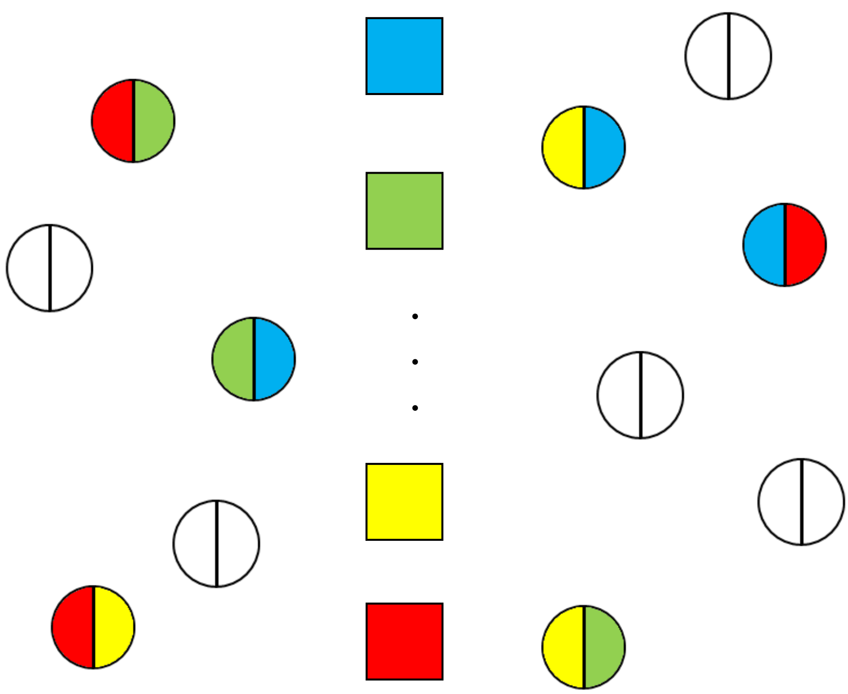


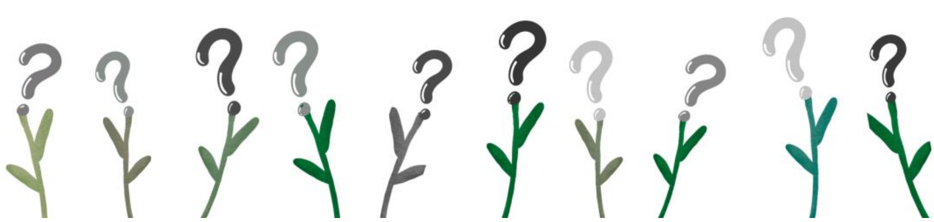
11.

Connect Many Double Dots

~*~

For which natural numbers *n* is it possible to connect *n* distinctly coloured squares to matching colours in all possible combinations of double coloured circles using those *n* colours without crossing any lines?



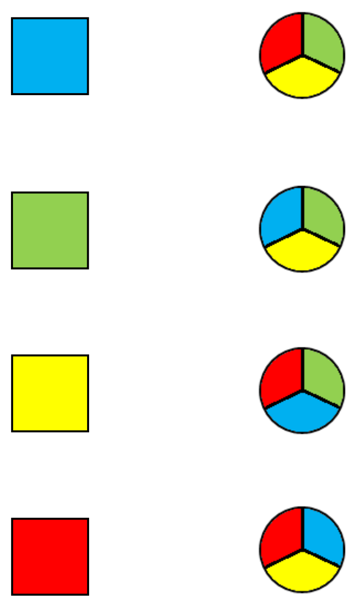


12.

Connect Triple Dots

~*~

In the following game of Connecting Dots, each circle has three colours. Connect each circle to all three squares that match its colours without crossing lines.

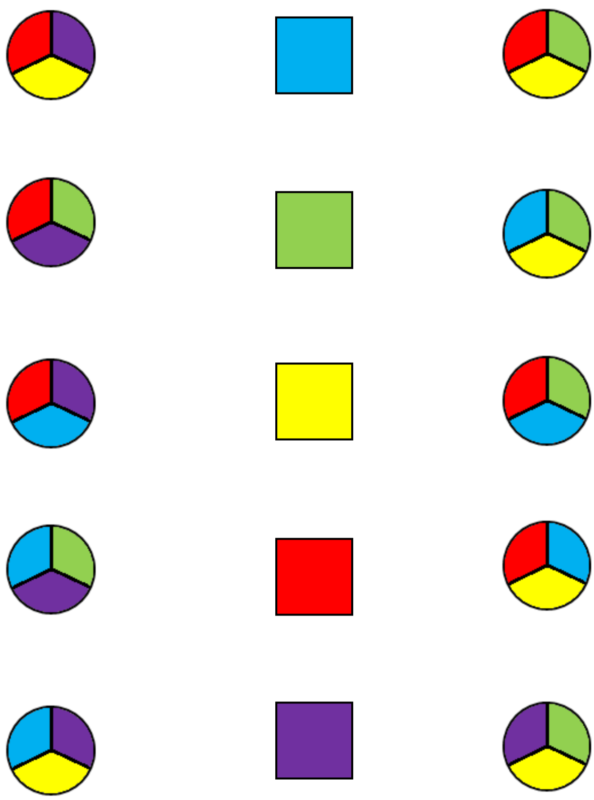


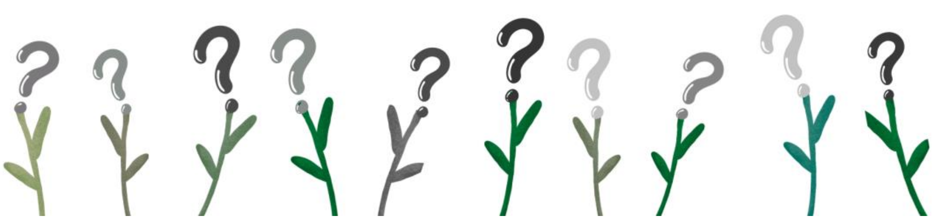
13.

Connect More Triple Dots

~*~

Again, connect each circle to all three squares that match its colours without crossing lines.



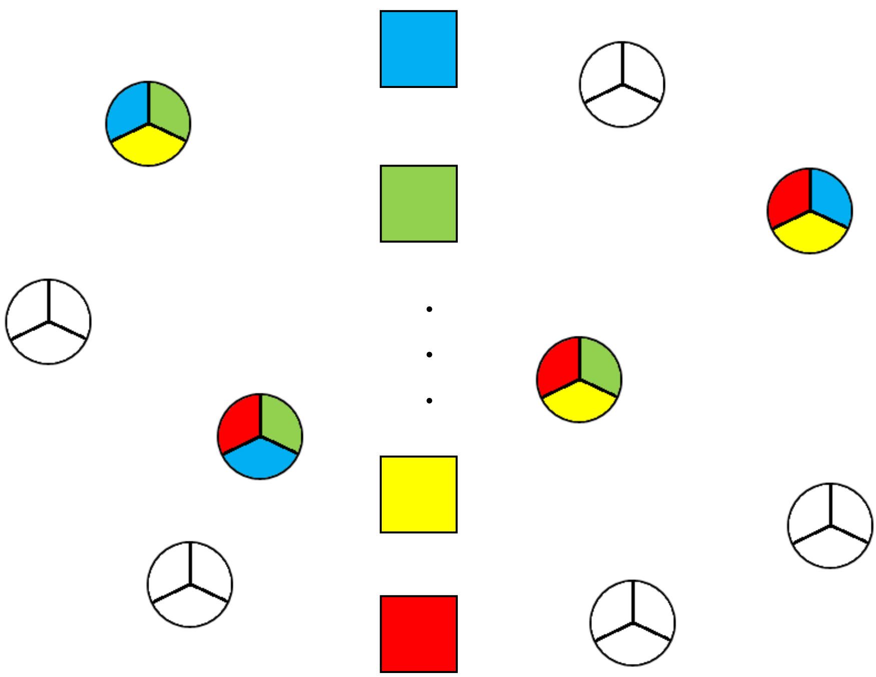


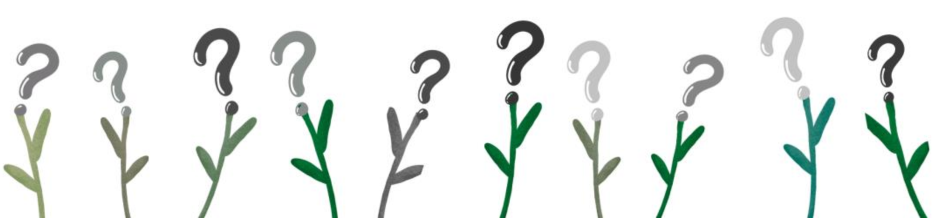
14.

Connect Many Triple Dots

~*~

For which natural numbers n is it possible to connect n distinctly coloured squares to matching colours in all possible combinations of triple coloured circles using those n colours without crossing any lines?





15.

Connect over Bridges

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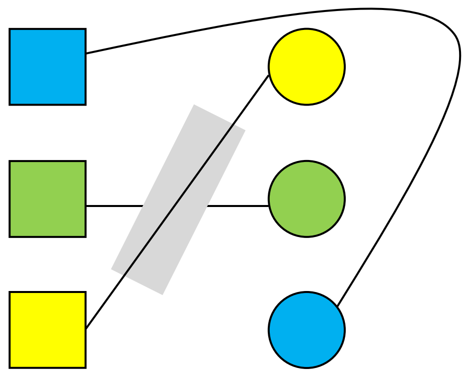
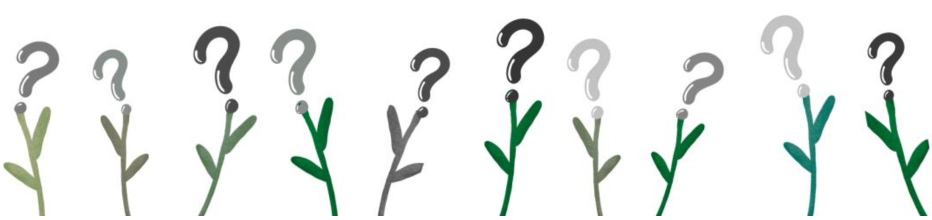
In the following solution to the game of Connecting Dots with three colours, the gray rectangle is a bridge that allows one line over another without touching.



If a bridge can be any length and be placed anywhere, what is the minimum number of bridges we need for a solution to Problem 13 in this book?

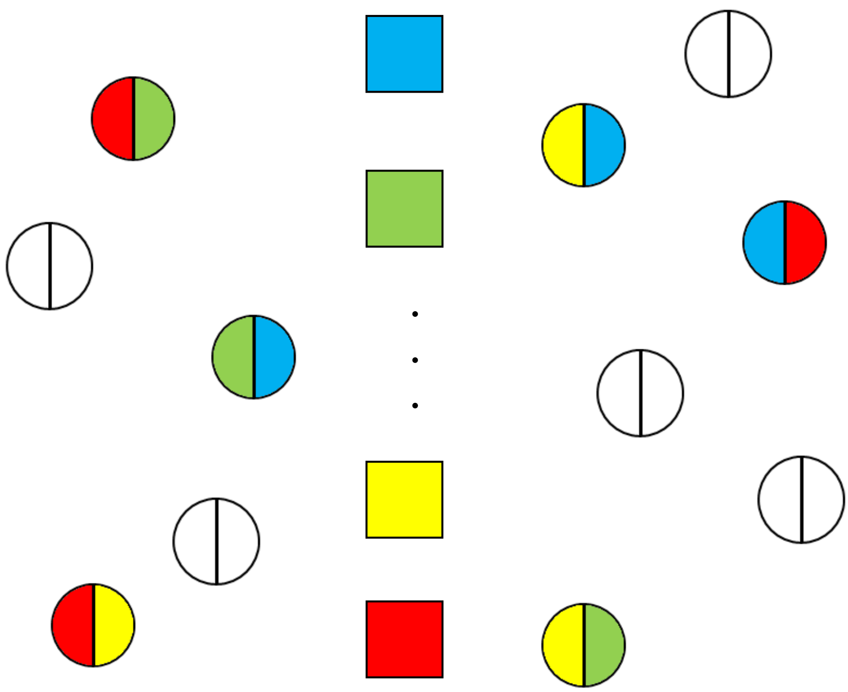


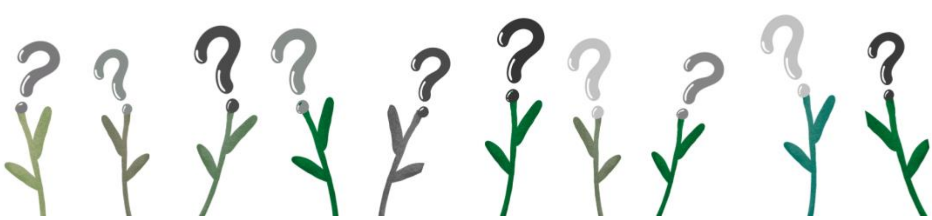
16.

Connect over Many Bridges

~*~

What is the minimum number of bridges we need to connect n distinctly coloured squares, where n is a natural number, to matching colours in all possible combinations of double coloured circles using those n colours without crossing any lines?





17.

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Make your own *easy* Connecting Dots problem:

18.

~*~

Make another *easy* Connecting Dots problem:

19.

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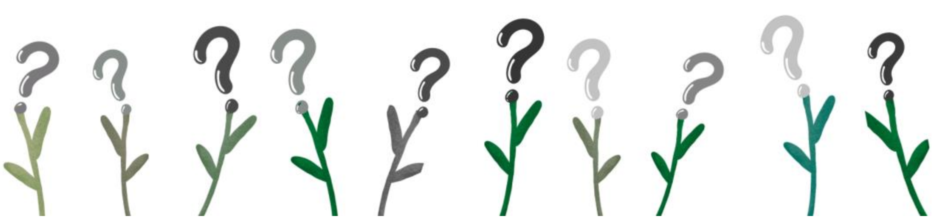
Make your own *hard* Connecting Dots problem:



20.

~*~

Make another *hard* Connecting Dots problem:

21.

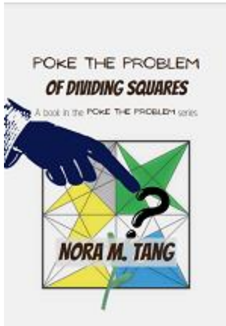
Tips

~*~

Here are some tips to help you get the most out of this book:

- Give yourself the space and time to explore each question properly. Don't give up when you don't know the answer at first glance. It takes time to wrestle with a good problem.
- Record your thoughts, ideas, and calculation results on a piece of scrap paper. This will help you see patterns and find new directions to try.
- If the answer to a question feels out of reach, or if you grow impatient, show it to a friend for help. Remember to also give them enough space and time to explore.
- Enjoy the process and don't fret about getting all the right answers. If you have wandered in the world of mathematics more than you would have otherwise, this book has served its purpose.

We hope you enjoyed this book as much as we did. Check out other books in the *Poke the Problem* series:



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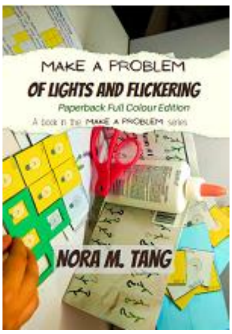

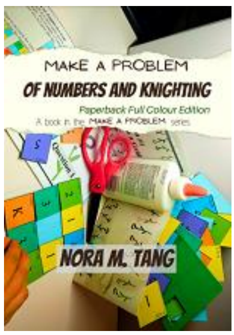

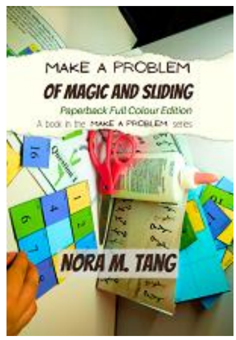

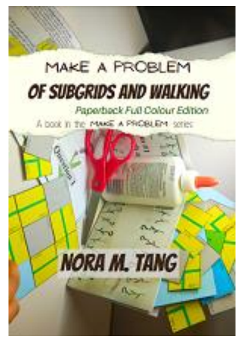

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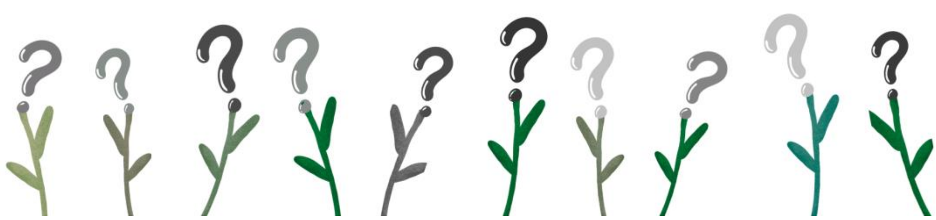

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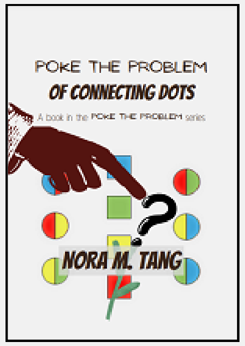

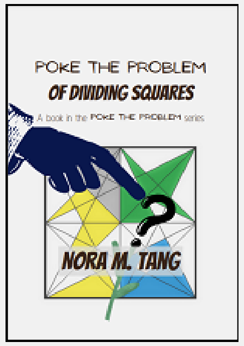

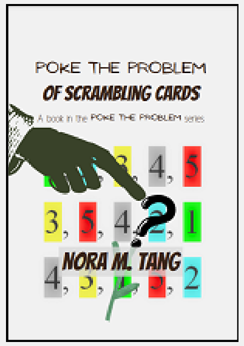

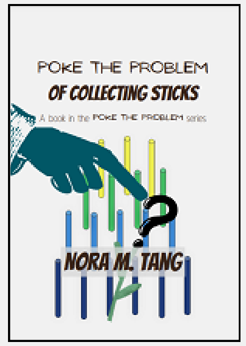

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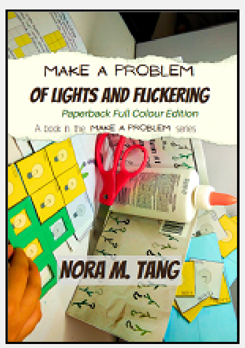

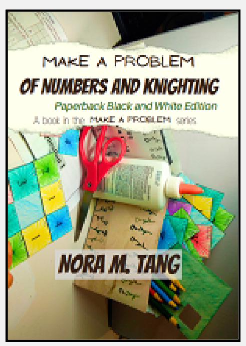

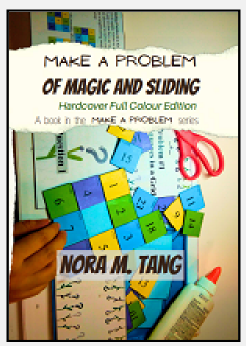

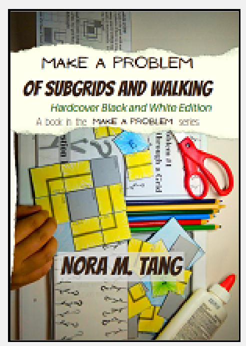

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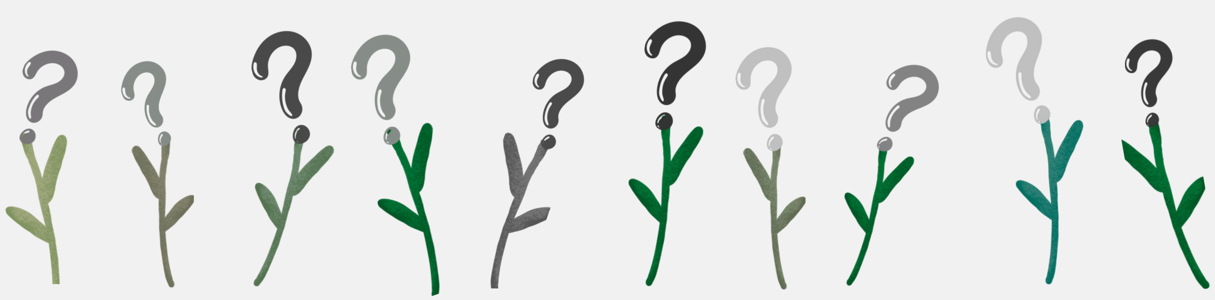
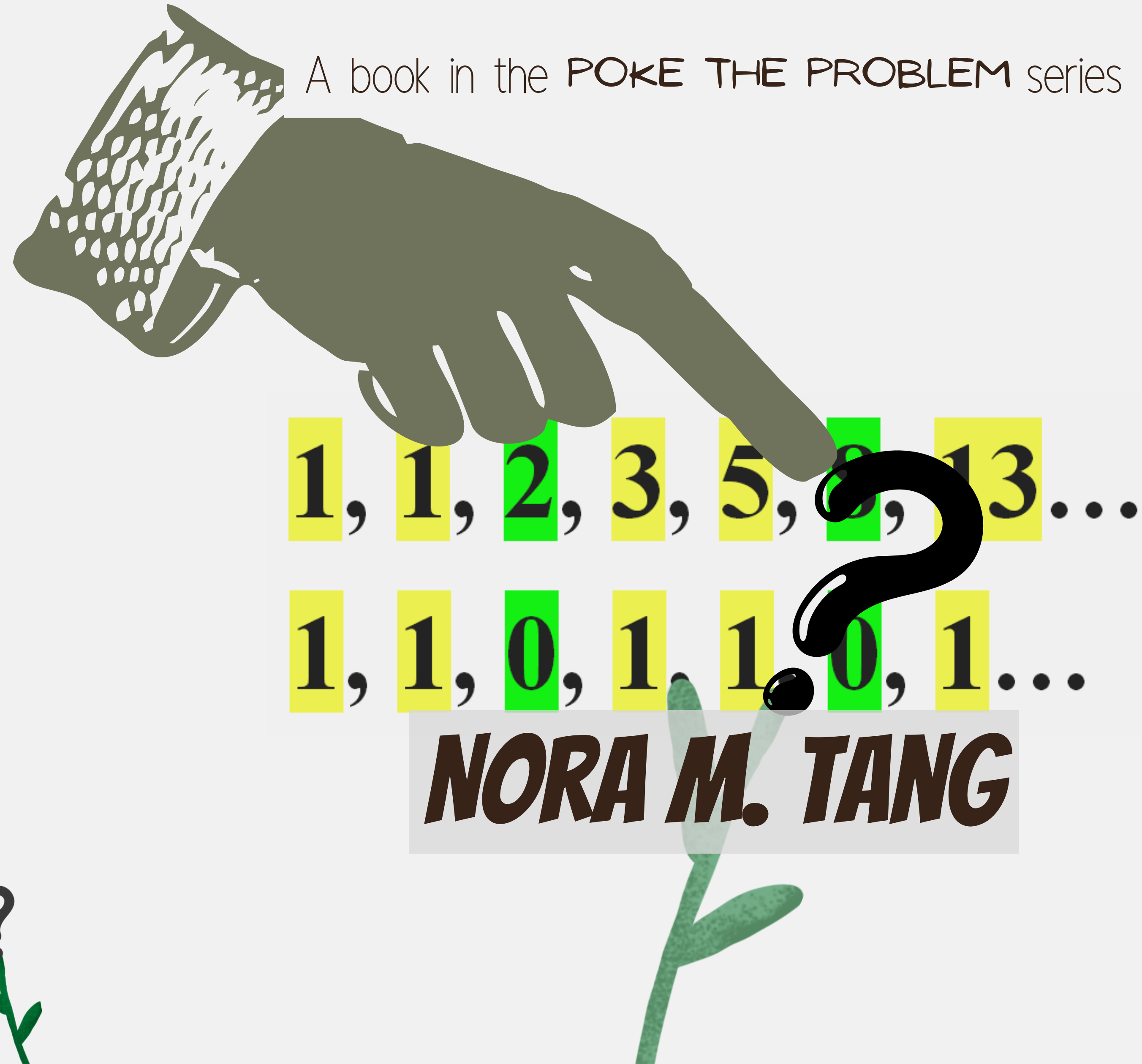


POKE THE PROBLEM

OF ADDING NUMBERS

A book in the POKE THE PROBLEM series



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ISBN-13: 978-1-989802-64-9

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Dedications:

For my daughters,

May you one day find yourselves in an elephant-less room surrounded by a boundless field of dandelions.

Table of Contents:

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It made me wonder where the math books are, not the stories about famous mathematicians, not the counting books or expository books about the number system (although that's a start, as alphabet books are for English), not the drilling workbooks for two digit additions and subtractions, not the repetitive brain teasers that all require the same three principles to solve, but genuinely interesting problems that make children wonder and explore and just *be* with math on their mind for a while.

Children are naturally curious, just as they are imaginative. Children can be excited by an interesting math question just as much as they can be by a story. Nor is there a lack of interesting problems math has to offer. Why, then, is there an absolute lack of interesting math problem books for children? More importantly, is this lack of exposure to interesting mathematics contributing to a child's relationship with the subject down the road?

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This book would not exist without the love and support of my husband, Victor, who has always done everything he can to hold space for my dreams and ideas, even when I couldn't see them clearly. Thank you for always being there for me.

Nora M. Tang

Revised April, 2024

0.

Before We Start

~*~

What you need to start enjoying problems in this book:

- a pen or pencil
- scrap paper
- a few minutes of free time
- a willingness to explore

There is *no time limit* for these problems. You are welcome to *use any calculator*. *The internet* and *other people's mathematical knowledge* are also within your allowed tools. You can even *change the question* if a different one seems more interesting to you, but be sure to record your new questions on the pages provided at the back of the book. In short, anything that helps you explore and understand is encouraged.

Mathematical knowledge needed to start exploring:

- multi-digit addition and division with remainders

Related mathematical topics to look up:

- divisibility, prime factorization, GCD and LCM
- proof by contradiction and induction
- Pisano periods and Lucas sequences

1.

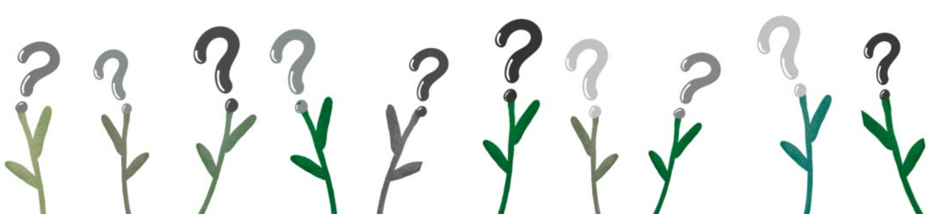
Find a Number

~*~

The *Fibonacci numbers* are a sequence of numbers following a recursive pattern (*recursive* meaning latter terms are calculated using previous terms). The sequence starts with 1 and 1, and every subsequent number is the sum of the two previous numbers, like this:

1, 1, 2, 3, 5, 8, 13…

Notice each number highlighted in green is the sum of the two previous numbers highlighted in yellow. Continue this pattern and find the 10^{th} number in the sequence.



2.

Check a Number

~*~

If the pattern for the Fibonacci numbers continue indefinitely, will 1000 be one of the numbers in the sequence?

1, 1, 2, 3, 5, 8, 13 … <u>1000</u> …

?

3.

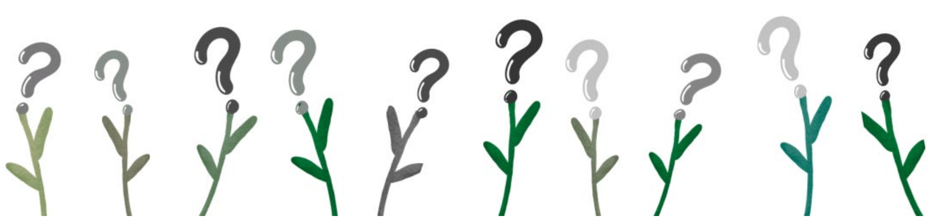
Pair a Number

~*~

Two natural numbers are said to be *relatively prime* (or co-prime) to each other if the only natural number divisor they share is 1. For example, 8 and 9 are relatively prime to each other. The divisors of 8 are 1, 2, 4, and 8, and the divisors of 9 are 1, 3, and 9. The only divisor in common between the two is 1.

Is there a pair of consecutive Fibonacci numbers (*consecutive* meaning neighbouring) that are *not* relatively prime to each other?

1, 1, 2, 3, 5, 8, 13…



4.

Modulate a Number

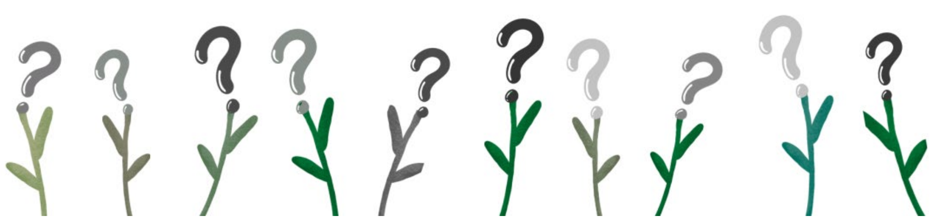
~*~

To *modulate* a number by 2 means to find the remainder when the number is divided by 2. If we modulate each number in the Fibonacci sequence by 2, this is the resulting sequence:

1, 1, 2, 3, 5, 8, 13…

mod 2: 1, 1, 0, 1, 1, 0, 1…

Notice the repeating pattern of two 1s and a 0. The pattern has a cycle length of 3 because it repeats every 3 numbers. Does this pattern continue indefinitely (*indefinitely* meaning endlessly)?



5.

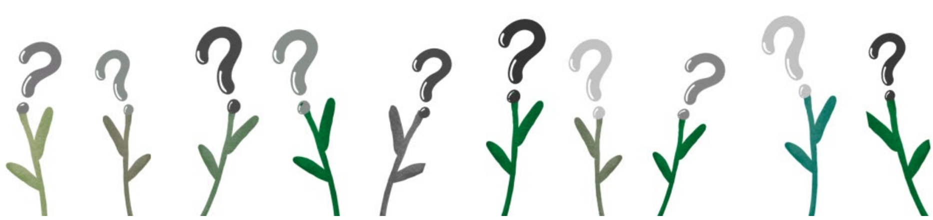
Cycle a Number

~*~

Find the cycle length of the Fibonacci sequence modulated by 3:

1, 1, 2, 3, 5, 8, 13…

mod 3: _, _, _, _, _, _, _…



6.

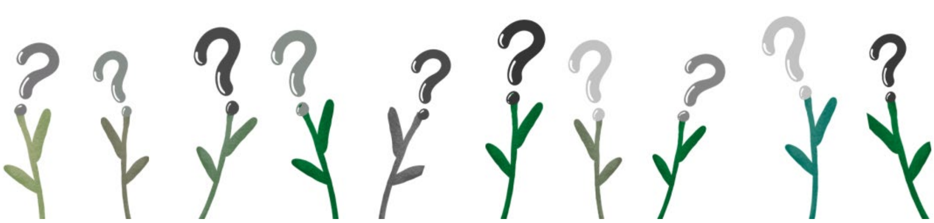
Check a Pattern

~*~

Is there a natural number *n* where modulating the Fibonacci sequence by *n* does not give a cyclical pattern?

1, 1, 2, 3, 5, 8, 13…

mod *n*: _, _, _, _, _, _, _…?



7.

Combine a Pattern

~*~

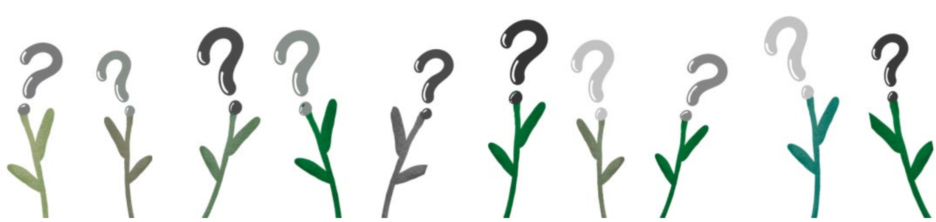
Write out the Fibonacci sequence modulated by 2 and by 3. Can you use those two sequences to write out the Fibonacci sequence modulated by 6?

mod 2: 1, 1, 0, 1, _, _, _…

mod 3: _, _, _, _, _, _, _…

mod 6: _, _, _, _, _, _, _…?

Find the cycle length of the Fibonacci sequence modulated by 6 using the information above.



8.

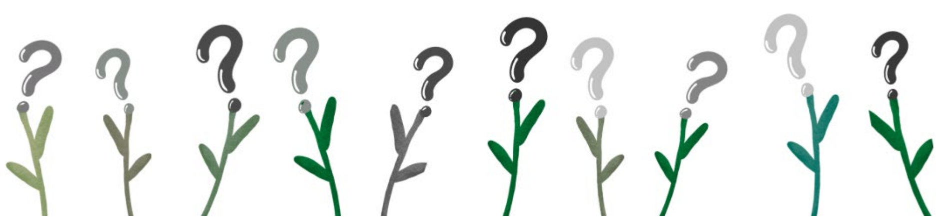
Combine More Patterns

~*~

If the Fibonacci sequence modulated by 5 has a cycle length of 20, what is the cycle length of the Fibonacci sequence modulated by 30?

mod 5: _, _, _, _, _, _, _…

mod 30: _, _, _, _, _, _, _…?



9.

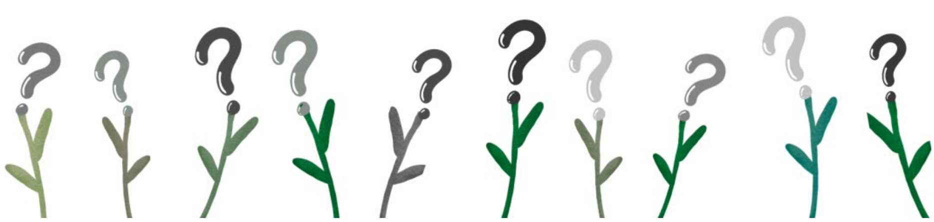
Find a Pattern

~*~

Given a natural number *n*, what is the cycle length of the Fibonacci sequence modulated by *n*?

1, 1, 2, 3, 5, 8, 13…

mod *n*: _, _, _, _, _, _, _…?



10.

Build a Number

~*~

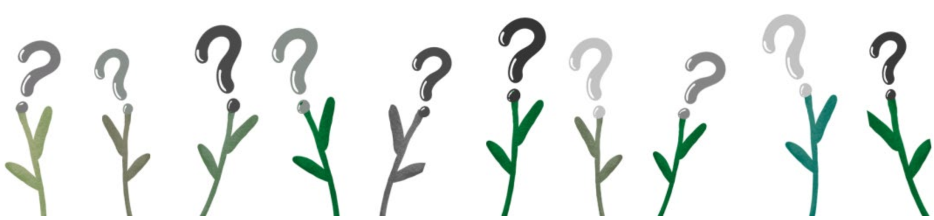
Write out all Fibonacci numbers smaller than 100 and use them to answer the question below:

1, 1, 2, 3, 5, 8, 13, ___, ___, ___...

How many ways are there to write 100 as the sum of distinct Fibonacci numbers (*distinct* meaning different from each other)?

100 = ___ + ___ + ___...?

100 = ___ + ___ + ___...?



11.

Build a Number Again

~*~

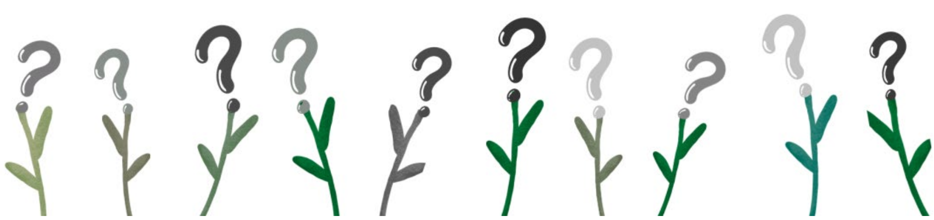
Here is one way you might have listed for the previous question. Notice that 2 and 3 are consecutive terms (or neighbours) in the Fibonacci sequence.

$$100 = 1 + 2 + 3 + 5 + 34 + 55$$

How many ways are there to write 100 as the sum of *non-consecutive* distinct Fibonacci numbers?

100 = __ + __ + __...?

100 = __ + __ + __...?



12.

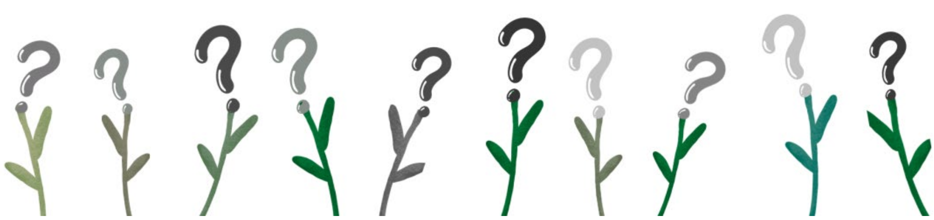
Build Any Number

~*~

Can we write any natural number *n* as the sum of non-consecutive distinct Fibonacci numbers?

1, 1, 2, 3, 5, 8, 13, ___, ___, ___...

***n* = ___ + ___ + ___...?**



13.

Ways to Build Any Number

~*~

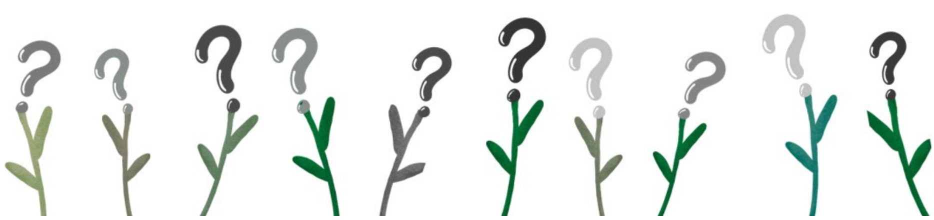
Given a natural number n, how many ways are there to write n as the sum of non-consecutive distinct Fibonacci numbers?

1, 1, 2, 3, 5, 8, 13, ___, ___, ___...

$$n = \_\_\_ + \_\_\_ + \_\_\_ \ldots ?$$

$$n = \_\_\_ + \_\_\_ + \_\_\_ \ldots ?$$

$$n = \_\_\_ + \_\_\_ + \_\_\_ \ldots ?$$



14.

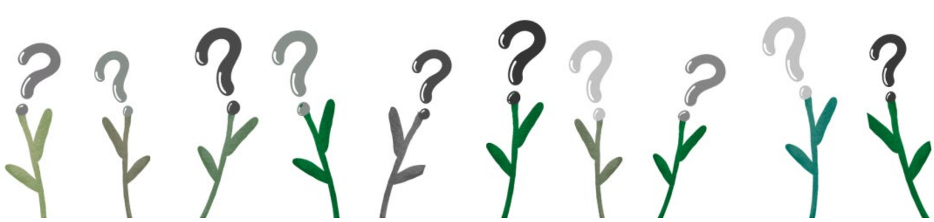
Add the Numbers

~*~

Write out the first 10 Fibonacci numbers. Then write out the *sum* of the first 4, 5, 6, 7, and 8 Fibonacci numbers. Compare the two lists of numbers.

1, 1, 2, 3, 5, 8, 13, ___, ___, ___...

Sums: 7, 12, ___, ___, ___...



15.

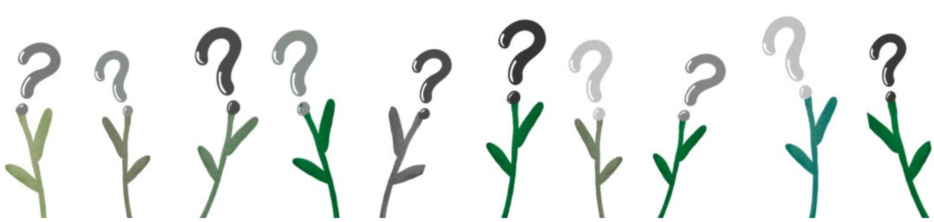
Add More Numbers

~*~

What pattern do you notice from the previous question? Does this pattern continue indefinitely (or endlessly)?

1, 1, 2, 3, 5, 8, 13, __, __, __...

Sums: 7, 12, __, __, __...



16.

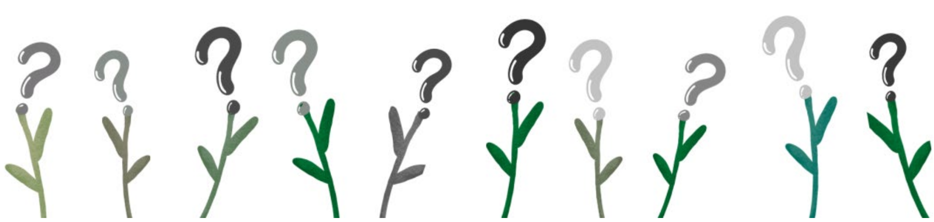
Try Other Numbers

~*~

The Lucas numbers follow the same recursive rule as the Fibonacci numbers except that the starting terms are 1 and 3 instead of 1 and 1. These are the first few Lucas Numbers:

1, 3, 4, 7, 11, 18, 29…

Try all previous questions using Lucas numbers instead of Fibonacci numbers.



17.

~*~

Make your own *easy* Adding Numbers problem:

18.

~*~

Make another *easy* Adding Numbers problem:

19.

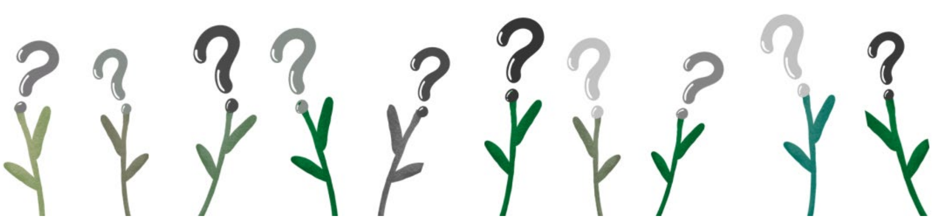
~*~

Make your own *hard* Adding Numbers problem:

20.

~*~

Make another *hard* Adding Numbers problem:



21.

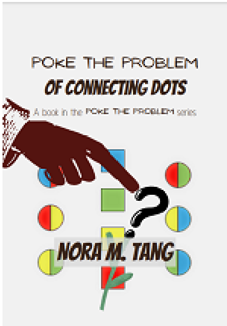
Tips

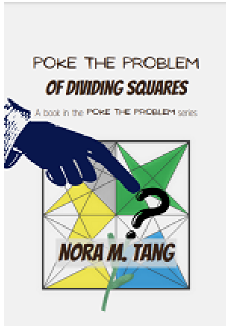
~*~

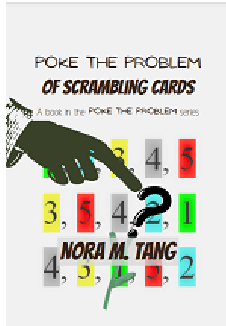
Here are some tips to help you get the most out of this book:

- Give yourself the space and time to explore each question properly. Don't give up when you don't know the answer at first glance. It takes time to wrestle with a good problem.
- Record your thoughts, ideas, and calculation results on a piece of scrap paper. This will help you see patterns and find new directions to try.
- If the answer to a question feels out of reach, or if you grow impatient, show it to a friend for help. Remember to also give them enough space and time to explore.
- Enjoy the process and don't fret about getting all the right answers. If you have wandered in the world of mathematics more than you would have otherwise, this book has served its purpose.

We hope you enjoyed this book as much as we did. Check out other books in the *Poke the Problem* series:







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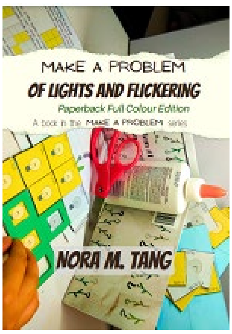

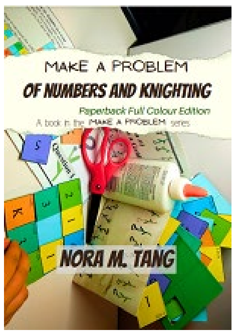

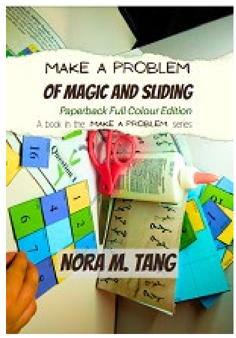

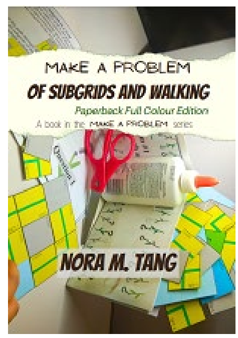

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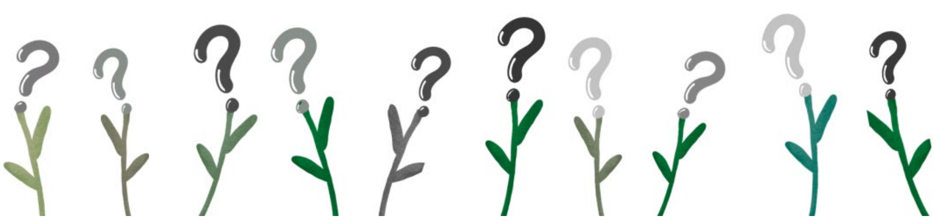
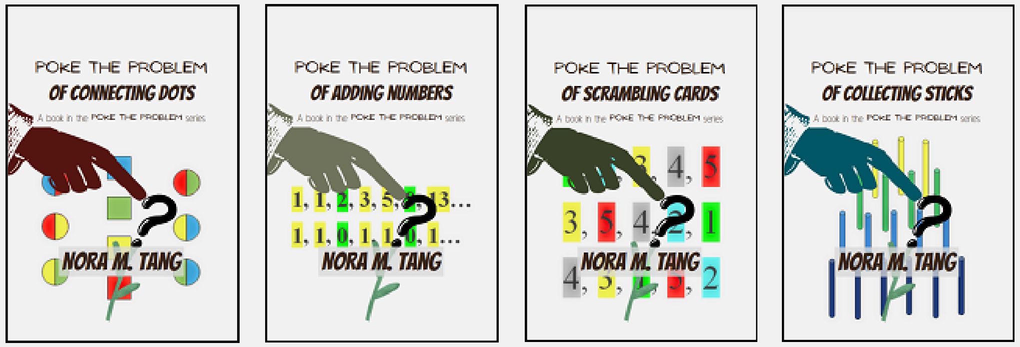


Check out other books in the

Poke the Problem *series:*



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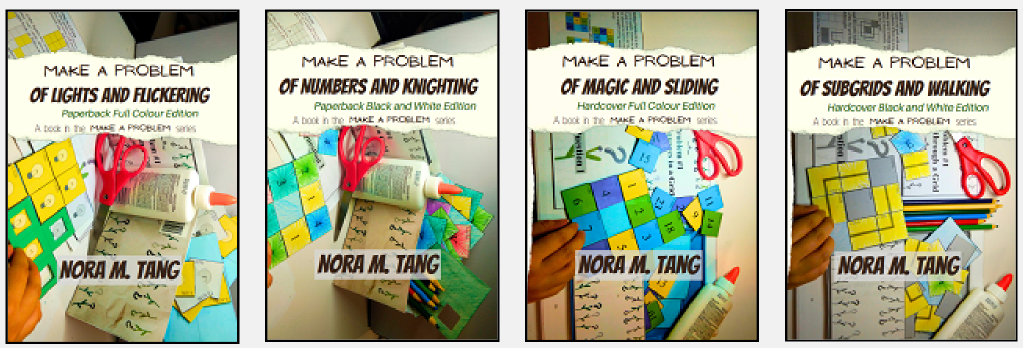

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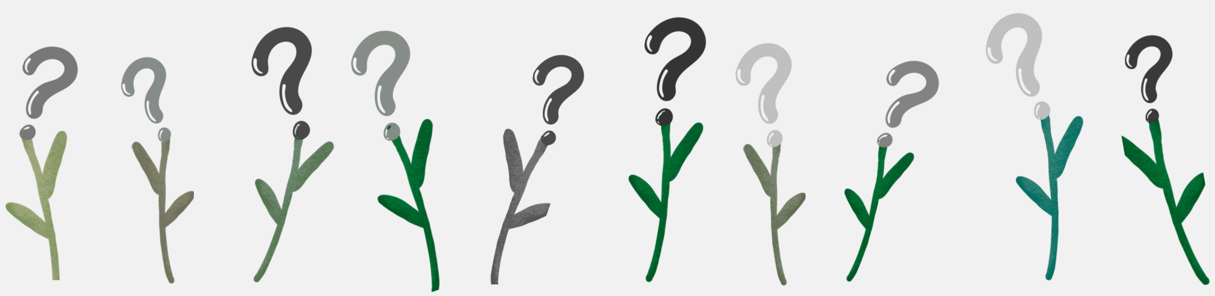
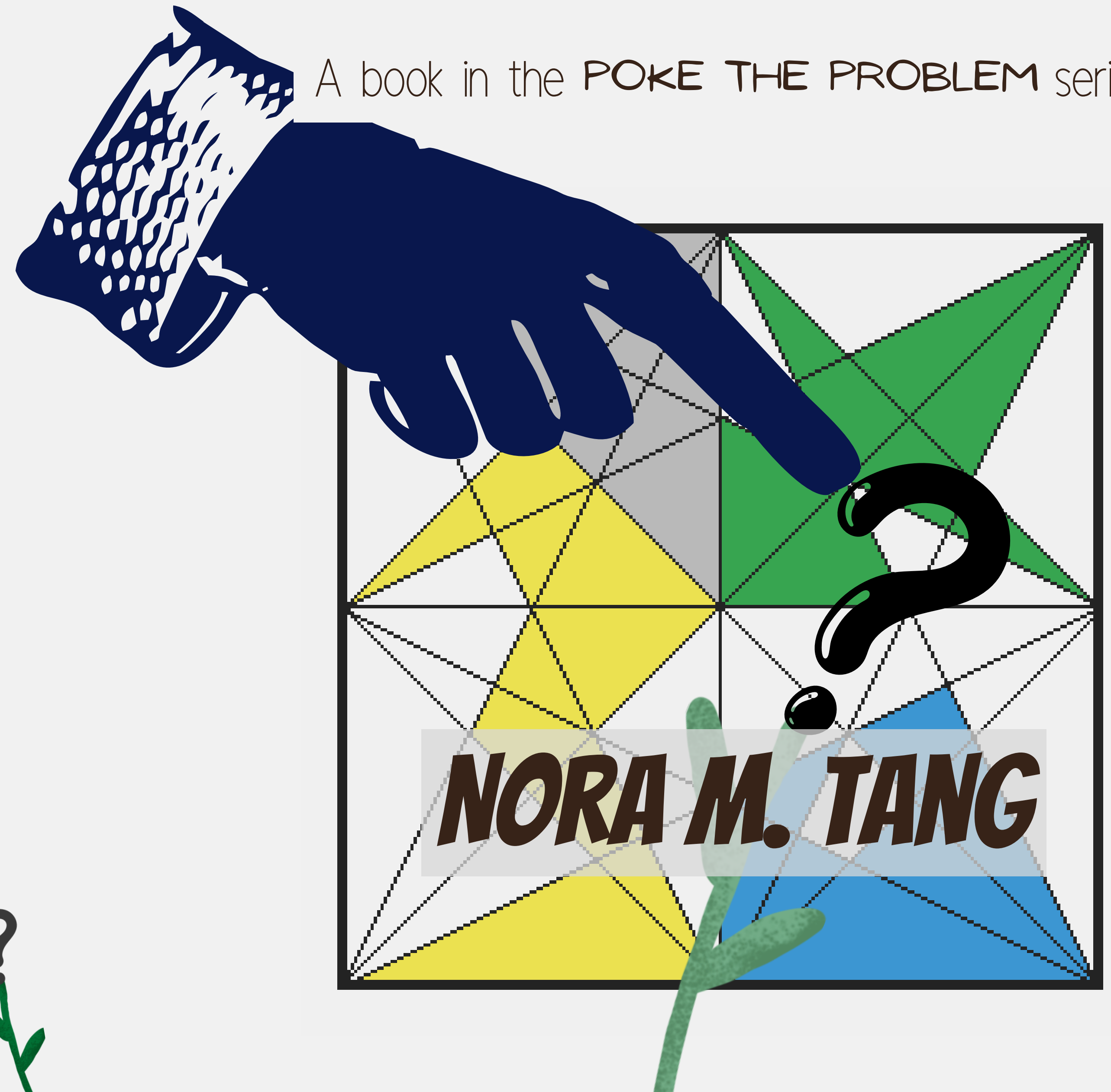


POKE THE PROBLEM

OF DIVIDING SQUARES

A book in the POKE THE PROBLEM series



Poke the Problem of Dividing Squares

A Book in the ***Poke the Problem*** Series

Nora M. Tang

ISBN-13: 978-1-989802-65-6

Dandelion Education

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Dedications:

For my daughters,

May you one day find yourselves in an elephant-less room surrounded by a boundless field of dandelions.

Table of Contents:

Preface

About a decade ago I had the chance to step back from my career in math education to start a family. From the time my young children were old enough to sit, I have been envious of language educators as children were surrounded by all the picture story books they may ever wish to read. The choices they have are endless, as are the lists of topics, styles, and authors to explore.

It made me wonder where the math books are, not the stories about famous mathematicians, not the counting books or expository books about the number system (although that's a start, as alphabet books are for English), not the drilling workbooks for two digit additions and subtractions, not the repetitive brain teasers that all require the same three principles to solve, but genuinely interesting problems that make children wonder and explore and just *be* with math on their mind for a while.

Children are naturally curious, just as they are imaginative. Children can be excited by an interesting math question just as much as they can be by a story. Nor is there a lack of interesting problems math has to offer. Why, then, is there an absolute lack of interesting math problem books for children? More importantly, is this lack of exposure to interesting mathematics contributing to a child's relationship with the subject down the road?

This series is one of our attempts at building an explorative mathematical experience for students. The problems in each book are inspired by the same scenario and build on each other, designed to orient the student in the scenario and encourage the student to explore and ask their own questions.

We offer no solutions in this book to remove the temptation on adults to turn this opportunity for discovery and wonder into a lecture, with the belief that it's always better for a child's relationship to mathematics to engage deeply with the problem and build their own structure of understanding than to be told what to memorize, even if it takes them longer to arrive at the same answers.

If the student requests a solution or a hint when working on these problems, we hope this solution or hint comes from an adult who is working on the problem with them, as the result of exploration and discovery, instead of being prepared by the problem-making authority. This way, the student learns about how to explore a

problem effectively, like how to set up clear record keeping schemes, how to avoid double counting or missing cases, how to spot patterns or ask the right questions to draw relevant conclusions. These are valuable skills of problem solving that are often omitted from a "complete solution" in the back of a book. By removing the authority from the source, we also ensure that the act of sharing a solution can become a joint effort of mathematical communication and verification instead of one-sided instruction.

We hope, of course, that this book would bring the love of math to more children and inspire them to take an interest in problem solving. But beyond that, we hope to see more adults join us in the effort of bringing more captivating math problems to children. We hope to see the day when children are surrounded by problems to try their hands on, just as they are surrounded by story books to read today. If the idea appeals to you, please consider joining our discord community (https://discord.gg/y83PNGXxhR) of authors, educators, and parents who want to bring more interesting math problem books to school aged children. We can help you start building your own in-class or in-home math problem book library or write and publish your own math problem books.

This book would not exist without the love and support of my husband, Victor, who has always done everything he can to hold space for my dreams and ideas, even when I couldn't see them clearly. Thank you for always being there for me.

Nora M. Tang

Revised April, 2024

0.

Before We Start

~*~

What you need to start enjoying problems in this book:

- a pen or pencil
- scrap paper
- a few minutes of free time
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There is *no time limit* for these problems. You are welcome to *use any calculator*. *The internet* and *other people's mathematical knowledge* are also within your allowed tools. You can even *change the question* if a different one seems more interesting to you, but be sure to record your new questions on the pages provided at the back of the book. In short, anything that helps you explore and understand is encouraged.

Mathematical knowledge needed to start exploring:

- areas of triangles
- fraction calculations

Related mathematical topics to look up:

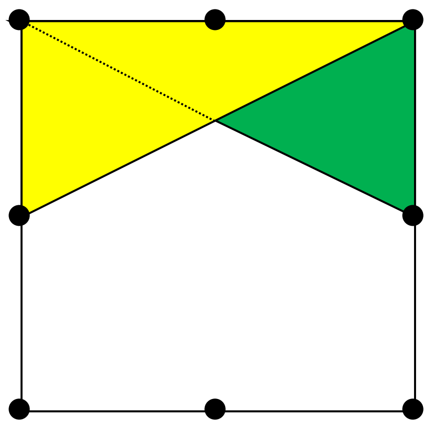
- lengths in similar figures
- visualization and volumes of 3D shapes

1.

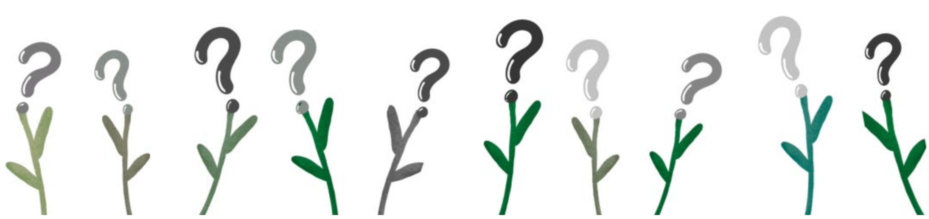
Divide the Square

~*~

Consider the midpoints of all sides and the corners of a square. We can divide the square into regions and calculate the fractional area of those regions. For example, the yellow region below has an area 1/4 of the square, and the green region has an area of 1/8 of the square.



Draw one region with an area of 1/3 of the square using straight lines among these points.

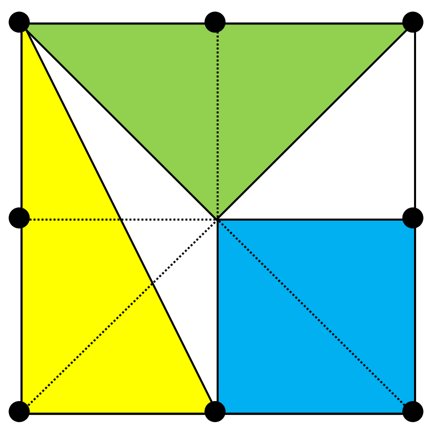


2.

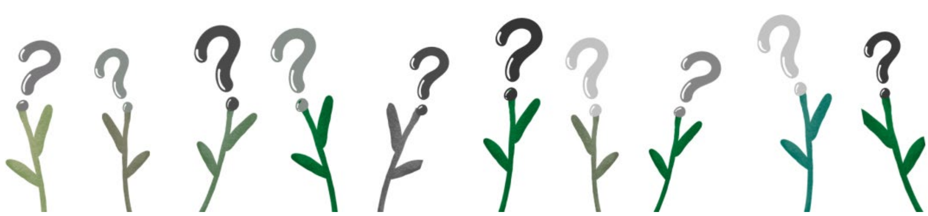
Draw the Regions

~*~

Notice that each of the coloured regions in this square has an area of a quarter, but they are all *non-congruent*, meaning they are differently shaped or sized, in this case just differently shaped because they share the same area:



How many non-congruent regions can be drawn using straight lines among the midpoint of sides and corners of the square and have an area of 1/3?

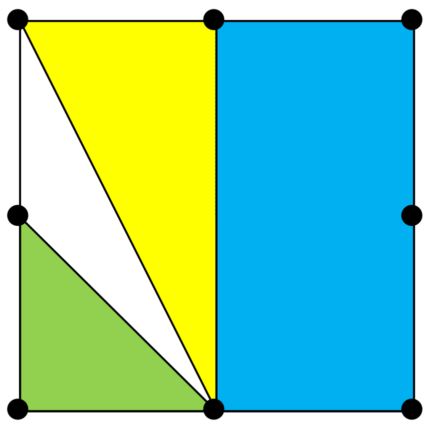


3.

Find the Areas

~*~

Notice that the coloured regions below have areas of 1/2, 1/4, and 1/8 of the square. These are *unit-fractional* areas, meaning the numerator of the fraction of the square they occupy is 1.



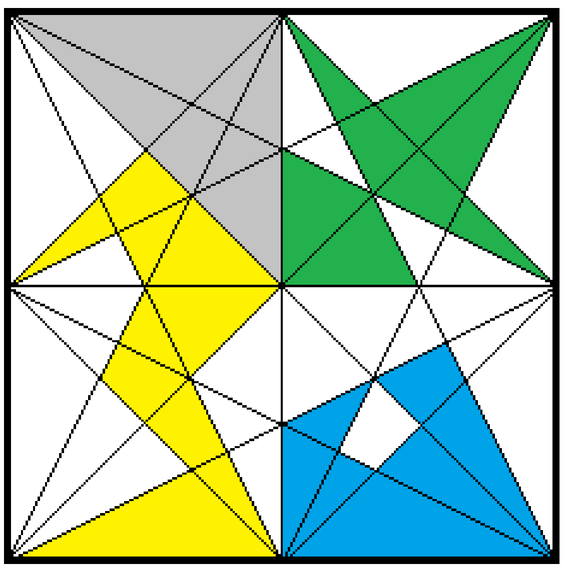
Find all unit-fractional areas we can make using straight lines among the midpoints of sides and corners of the square.

4.

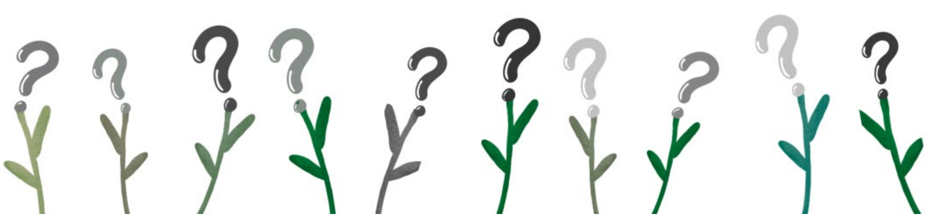
Count the Regions

~*~

The square below is divided using all straight lines among midpoints of its sides and its corners. Let's count the regions that are *connected*. For example, the portion shaded yellow and the portion shaded blue are both regions. The green portions are disconnected and therefore not a region.



How many non-congruent (or differently shaped or sized) regions are there in the gray portion alone?

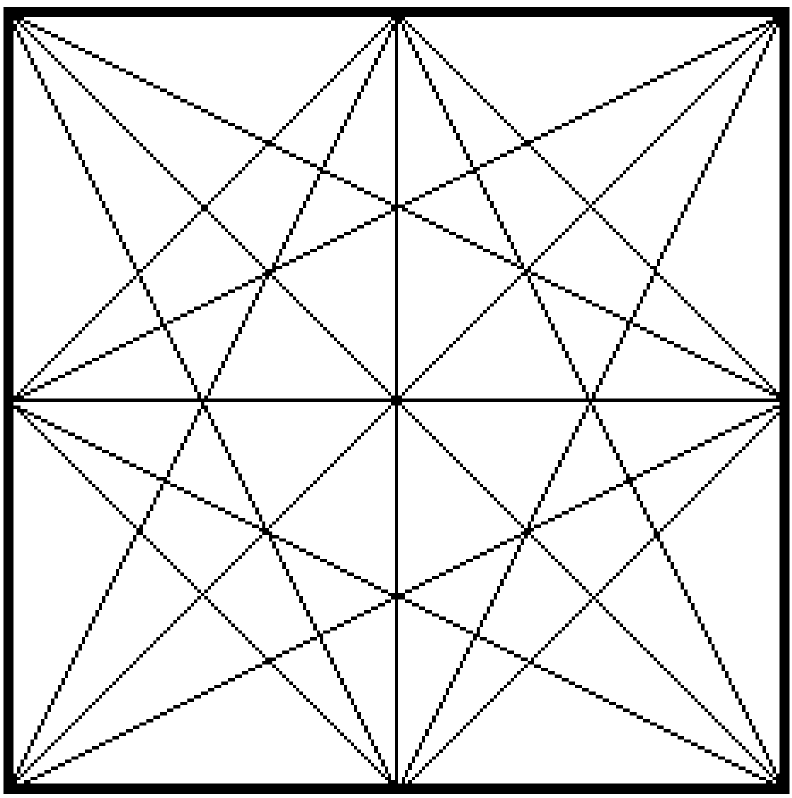


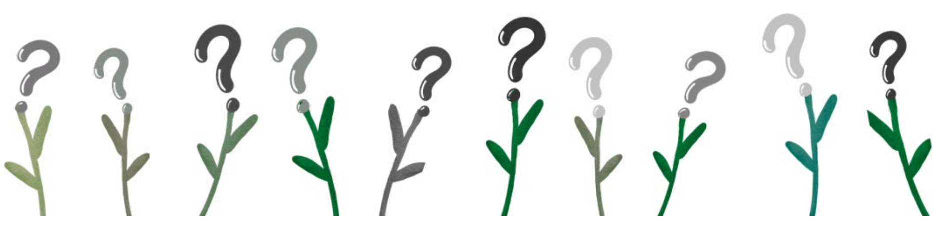
5.

Count More Regions

~*~

If we count regions in the entire square using the same lines and the same rules as the previous question, how many non-congruent (or differently shaped or sized) regions of any size are there in the square?



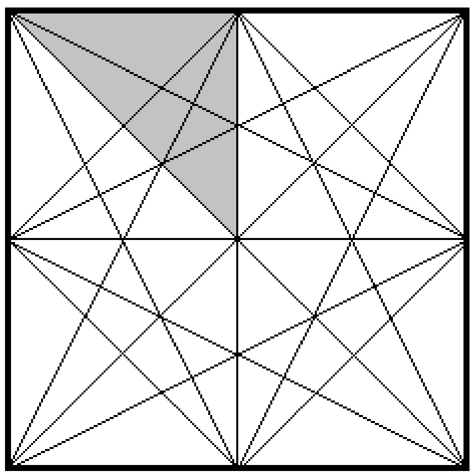


6.

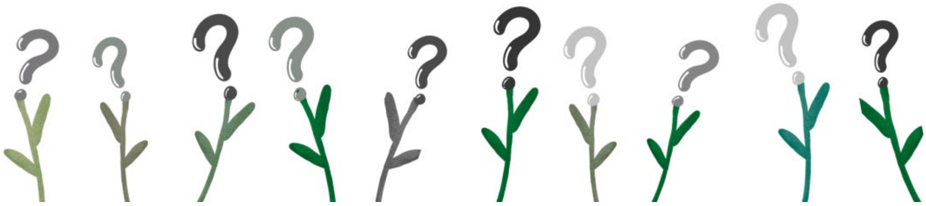
Count the Pieces

~*~

Notice that by drawing lines among midpoints of sides and corners, the square can be divided into 56 individual pieces. There are 7 in the gray wedge and 8 identical wedges in the square.



How many individual pieces can we divide the square into if each side is divided into three equal parts instead of two?

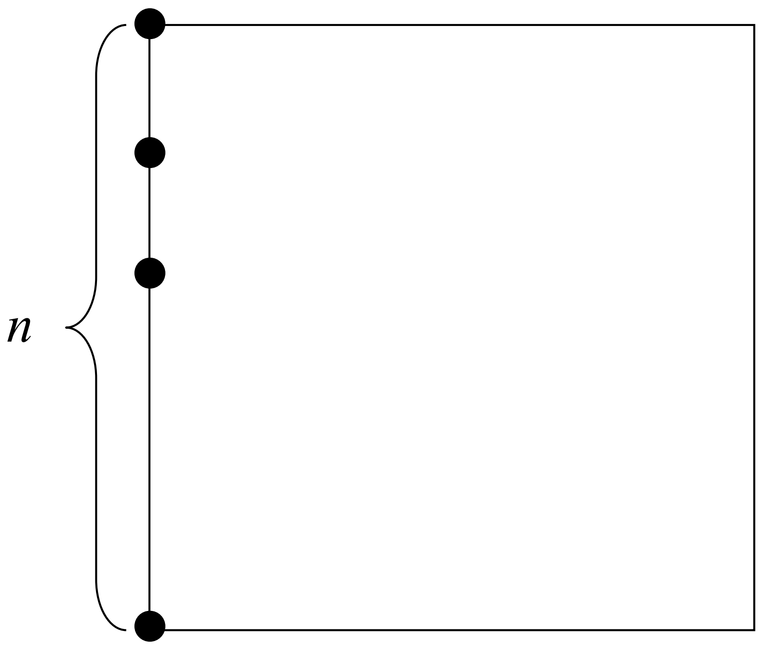


7.

Count Many Pieces

~*~

How many individual pieces can we divide the square into if each side is divided into n equal parts, where n is a natural number?

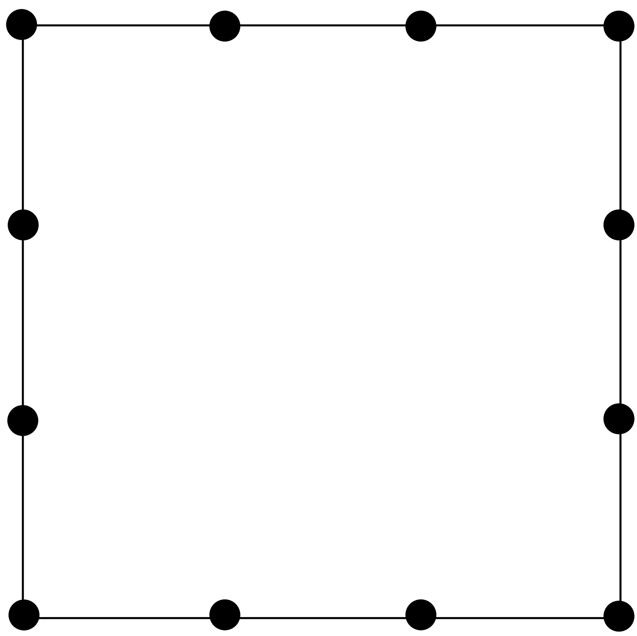


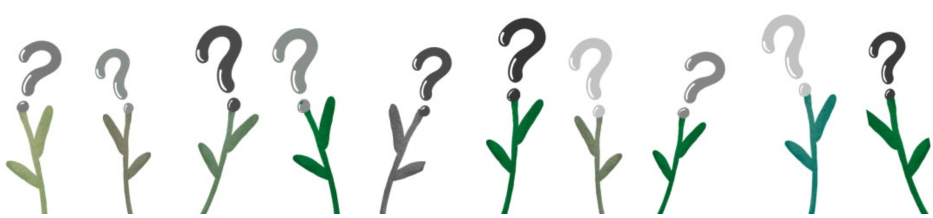
8.

Find More Areas

~*~

Which *unit-fractional* areas (areas with 1 as the numerator) can be drawn using straight lines among points that divide each side of the square into thirds and corners?



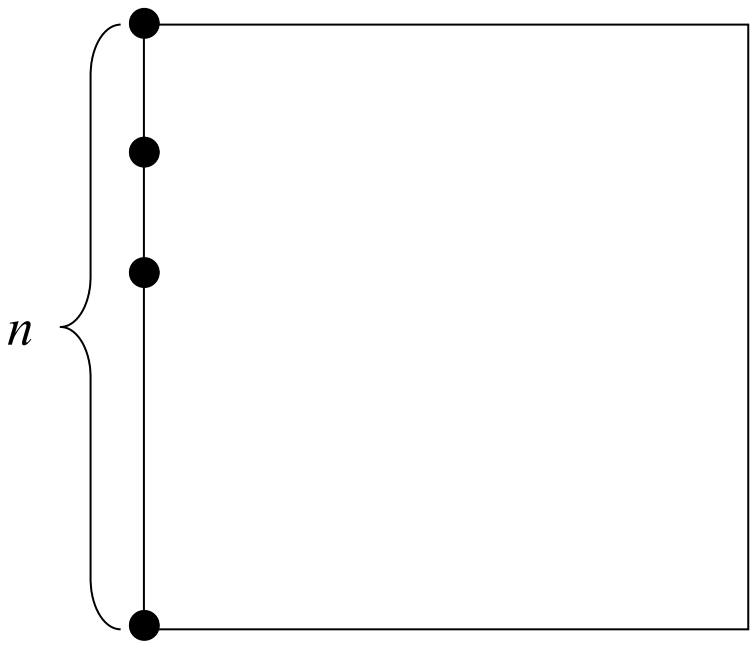


9.

Find Many Areas

~*~

Which *unit-fractional* areas can be drawn using straight lines among corners and points that divide each side of the square into n equal parts, where n is a natural number?

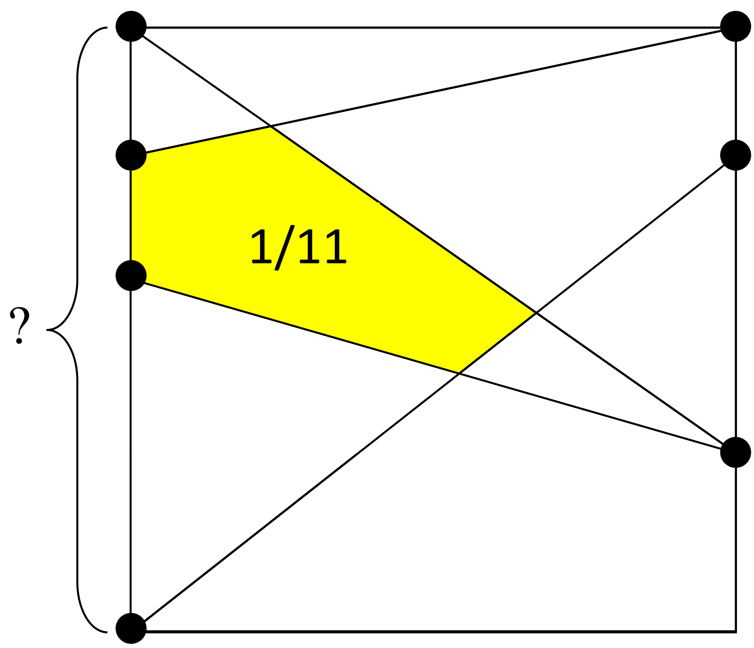


10.

Divide the Side

~*~

What’s the smallest number of equal parts we need to divide each side into to get a region with an area of 1/11 of the square?

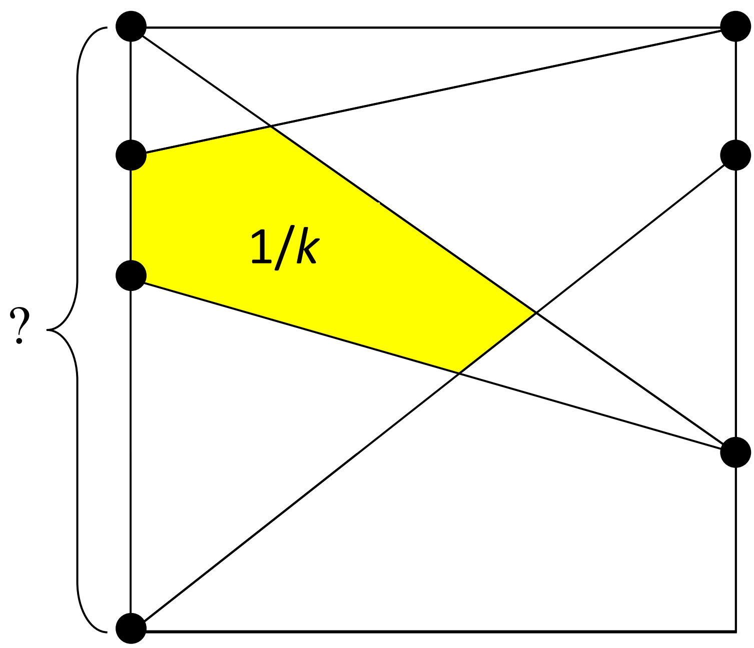


11.

Divide the Side Again

~*~

What's the smallest number of equal parts we need to divide each side into to get a region with an area of $1/k$ of the square, where k is a natural number?

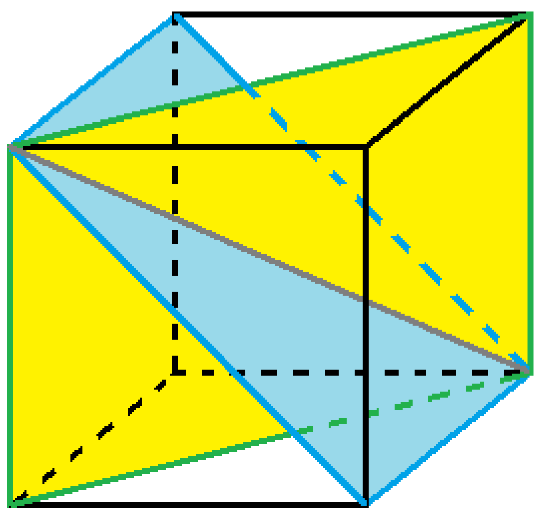


12.

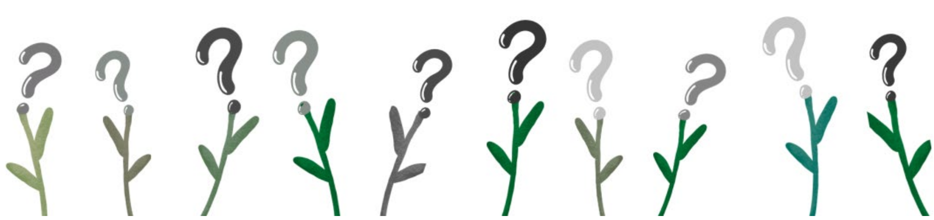
Cut the Cube

~*~

Consider a cube whose edges are undivided (because it gets complicated quickly in 3D). Two edges in 3D are called *coplanar* if they can be placed on the same plane. For example in the cube below, the top left and bottom right blue edges share a plane coloured in blue.



If we cut the cube using the plane of every pair of coplanar edges, what are the volumes of each of the resulting pieces?

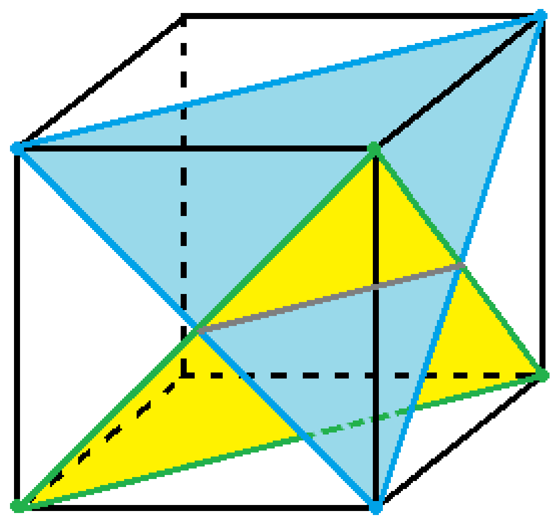


13.

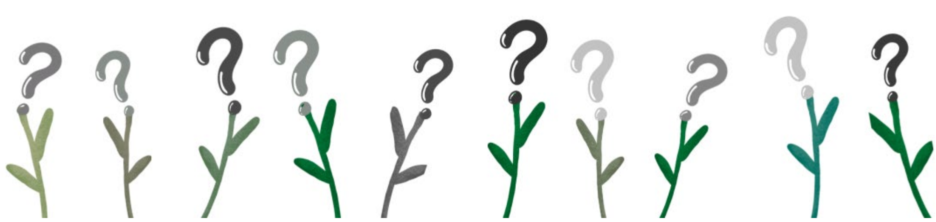
Cut the Cube More

~*~

A set of points in 3D are called *coplanar* if they can all be placed on the same plane. For example in the cube below, the three green vertices share a plane coloured in yellow. Notice that the four vertices on each cutting plane in the previous question are also coplanar.



If we cut the cube using the plane of every set of coplanar vertices, what are the volumes of each of the resulting pieces?

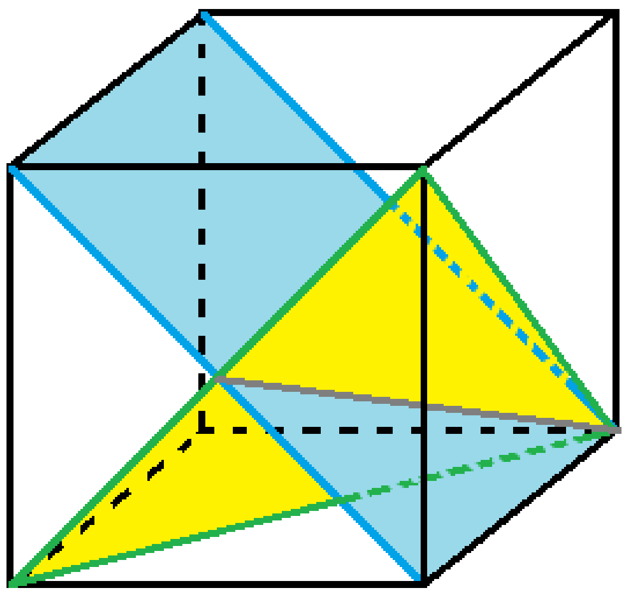


14.

Find the Volumes

~*~

What *unit-fractional* volumes (volumes with 1 in the numerator) of a cube can we get if we cut the cube using the plane of every set of coplanar vertices?

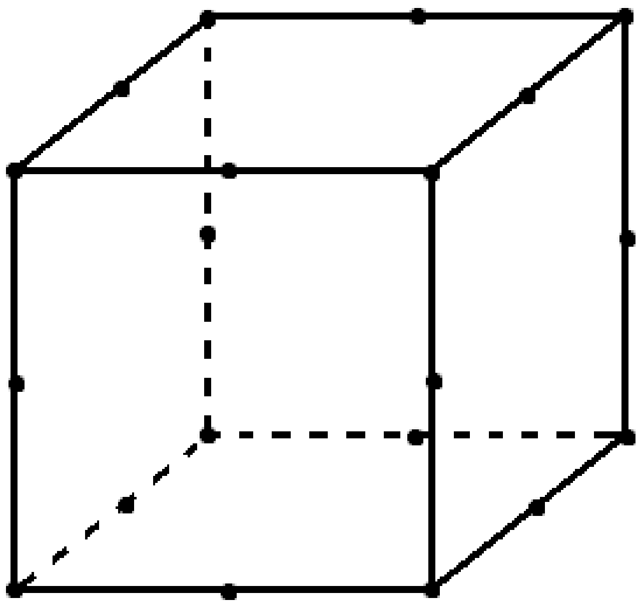


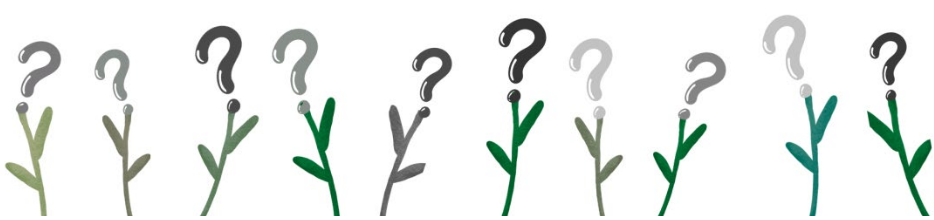
15.

Count 3D Pieces

~*~

How many individual pieces can we divide the cube into if we cut the cube using the plane of every set of coplanar points among the vertices and midpoints of sides?



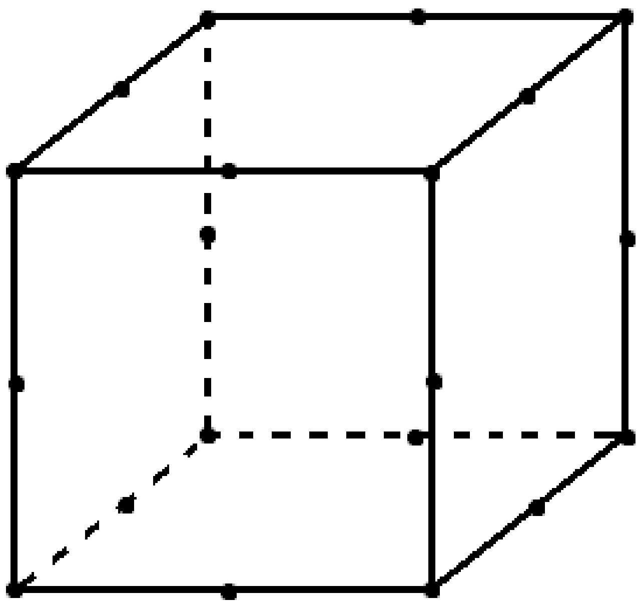


16.

Find More Volumes

~*~

What unit-fractional volumes of a cube can we get if we cut the cube using the plane of every set of coplanar points among the vertices and midpoints of sides?



17.

~*~

Make your own *easy* Dividing Squares problem:

18.

~*~

Make your own *easy* Dividing Cubes problem:

19.

~*~

Make your own *hard* Dividing Squares problem:

20.

~*~

Make your own *hard* Dividing Cubes problem:

21.

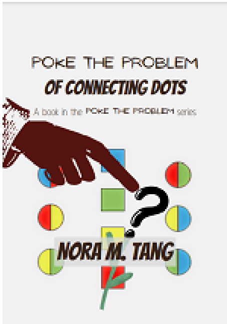
Tips

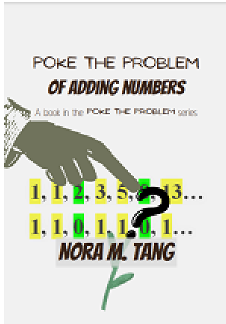
~*~

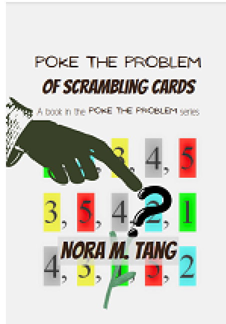
Here are some tips to help you get the most out of this book:

- Give yourself the space and time to explore each question properly. Don't give up when you don't know the answer at first glance. It takes time to wrestle with a good problem.
- Record your thoughts, ideas, and calculation results on a piece of scrap paper. This will help you see patterns and find new directions to try.
- If the answer to a question feels out of reach, or if you grow impatient, show it to a friend for help. Remember to also give them enough space and time to explore.
- Enjoy the process and don't fret about getting all the right answers. If you have wandered in the world of mathematics more than you would have otherwise, this book has served its purpose.

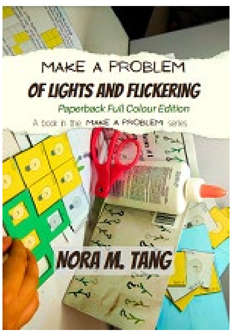
We hope you enjoyed this book as much as we did. Check out other books in the *Poke the Problem* series:

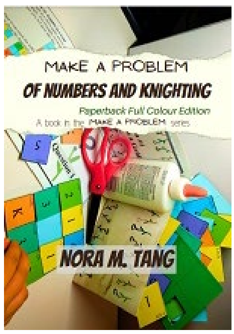


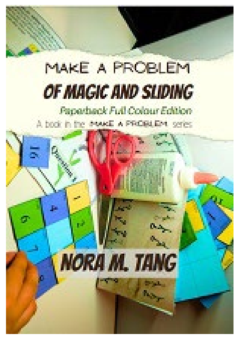


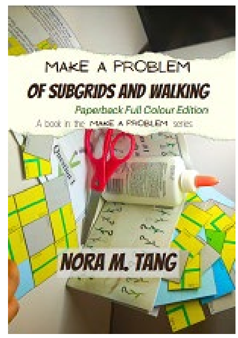


Also check out books in our *Make a Problem* series:









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